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LABORATORY SIMULATION OF A GEOMORPHOLOGICAL RUNOFF ROUTING MODEL USING LIQUID ANALOG CIRCUITS

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The complexity of the rainfall-runoff process has caused various models to be developed for flood routing. A liquid analog model (LAM) is introduced as a new hydrological laboratory device. The scaling and construction of this model in the laboratory are described. The result of the model application for laboratory simulation of a conceptual geomorphological runoff routing model, using real watershed rainfall-runoff data, is extensively discussed. This model can be easily used for simulation of other flood routing problems in the laboratory. Due to simple structure, visible properties and convenient operation of the LAM, it can be reliably considered as an educational instrument in modeling rainfall-runoff problems.

INTRODUCTION

It is conceded by all the experts of hydrology that the process of rainfall running off a natural watershed is a very complex process and it is not incorrect to assume that this concession is unanimous. In addition, it is also expressed that this process is ill defined over time, i.e. the runoff changes from one season or storm to another. Any stimulus or input to a complex and ill defined natural system, such as a watershed will naturally produce a complex response or output. On the complexity of watershed hydrology, Mulligan (2004) states "there are many areas of hydrology where our understanding of the processes is basic but still sufficient to develop models; but there are still areas in which the complexity of hydrological processes is so great or the information so little in which there is still much progress to be made". The rainfall-runoff process can be considered as such an area. At the present it seems difficult to give an exact mathematical description to this complicated process; however in many cases there is no necessity to take recourse to a complete description of the phenomenon at all stages. This is because the initial data and all the elements of the process can be obtained only with a known approximation and with a rough schematization in time and space. In this way the art of a successful modeler is to create simple models that can successfully and reliably describe such a complex process; in other words models are devices and methods used to form simplicity out of complexity (Clarke, 1973). To achieve this purpose numerous hydrological models have been developed. In Figure 1 a chart of hydrological models which was presented by Fleming (1975) and modified by the authors is shown. The boundary of each model is not absolute and may overlap the others. Recently in spite of the progress in flood routing computer models which have their own advantages and disadvantages (Nourani and Mano, 2005), the physical models in the forms of iconic and analog models are still used by researchers, especially for laboratory and educational purposes (Singh, 1988).

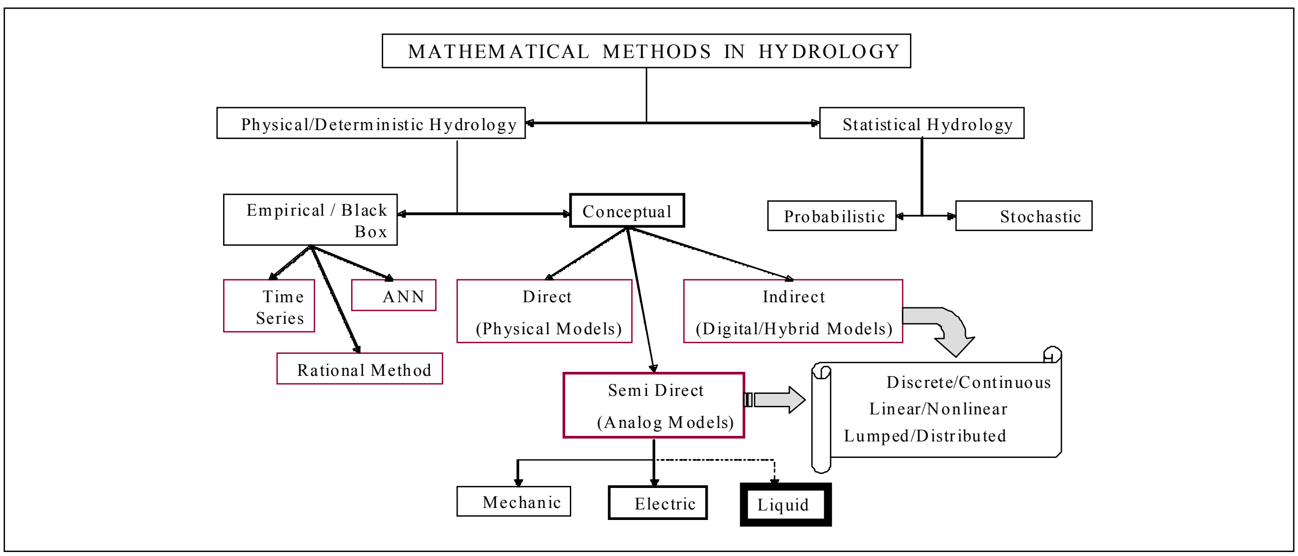

Figure 1. Hydrology models.

Due to the complexity of the rainfall-runoff process and the absence of data with which to describe in detail the character of heterogeneous watersheds and of spatially distributed inputs, simulation of the rainfall-runoff process is generally based on conceptual models. Such models are built from a concept of the functioning of the studied real system and contain some parameters that must be estimated. One of the main subdivisions of the conceptual models is the linear reservoir model that assumes the outflow is directly proportional to the storage i.e. (Chow, 1964):

$$S=KO \quad (1)$$

in which *O, S* and *K* are output discharge, reservoir volume and storage coefficient respectively. In this model a watershed or channel is represented by a series or parallel configuration of linear reservoirs with constant or variable storage coefficients. By routing water through these reservoirs, a unit hydrograph can be obtained. As a basic study, Nash (1957) proposed a conceptual model by representation of a watershed as a series of identical linear reservoirs in a cascade form and derived a mathematical equation for the instantaneous unit hydrograph (IUH). Because some streams have not been gauged and lack observed hydrological data, for such data-poor basins it is required that the model parameters be identified by using basin physical characteristics rather than calibration. In this way, Boyd (1978) and Boyd et al. (1979) developed a storage routing method based on catchment geomorphology. With a similar methodology, Karnieli et al. (1994) and Hsieh and Wang (1999) developed other geomorphological runoff routing models. Rodriguez-Iturbe and Valdes (1979) and Gupta et al. (1980) presented the GUH (Geomorphological Unit Hydrograph) on the basis of the assumption of exponentially distributed waiting time of the drop in a stream of given order, where the channel network and drainage areas are described through Horton relations and the Strahler ordering scheme. Recently many semi-distributed models have been established which usually use DEM (Digital Elevation Model) as input data. A list of these models such as TOPMODEL (Beven and Kirkby, 1979), is mentioned by Nourani and Mano (2005).

Regarding the utilization and use of the analog models which are used in hydrological modeling as shown in Figure 1, Jackson (1968) and Quick (1965) applied respectively electric and mechanic analog models to routing problems. Shen (1965) described the use and results of a experimental research with electric analog systems for modeling flood routing in some fictional watersheds. Sokolosky and Shiklomanov (1969) and Levin (1969) used an electric analog model for simulation of Nash's model. More recent use of electric analog models in combination with various digital methods has clearly extended hydrological models perception (Abedini, 1998).

The objective of this paper is to introduce a special kind of analog model which uses liquid flow in contrast to electric flow. The liquid analog model and its governing equations, the model components, scaling and its setup in the laboratory are depicted. The application of this new laboratory model to simulate Boyd's geomorphological runoff routing model for a real watershed, in order to determine the outflow of watershed, is described. Finally the utility of this instrument, based on the obtained results in the laboratory, and its educational and research benefits are explained and discussed.

LIQUID ANALOG MODEL AND GOVERNING EQUATIONS

Two systems are analogs of each other if the governing equations of one system are similar to the other. Therefore the solution of one system can be applied to the other by proper scaling.

The Liquid Analog Model (LAM) was patented by one of the authors (Monadjemi, 2001). Like electrical and mechanical analog models which are based on Kirchhoff and Newtons' laws respectively, a liquid analog model is constructed based on continuity and Darcy's law. Each liquid analog system consists of at least one circuit and each circuit has three major components: a reservoir element, a friction element and a constant head overflow device. These elements are connected using relatively large diameter pipes to ensure that the flow regime in the pipes remains laminar. The reservoir element is graduated to facilitate the reading of liquid head at any time. Although any kind of liquid can be used in this circuit, water is chosen because of its accessibility and easy operation.

Liquid and electric circuits are presented in Figures 2-a and 2-b; it will be shown they are analogous to each other. The reservoir and the friction elements in the liquid circuit have the same roles as the capacitance and resistance in the electric model respectively. Figure 2-c shows a friction element of the LAM; this element is built on the basis of water flow through a porous medium (Darcy's Law). The friction element, in the form of a cylindrical tube, is filled with porous media such as sand with a hydraulic conductivity of *c*. The friction element has a length of *l* and a sectional area of *a.* The two ends of the tube are packed by coarse gravel and two screens are also used to separate the sand and the gravel. Because of the pipes shortness and low flow velocity in them, the friction and minor head losses are negligible. By applying Darcy's law, in the laminar flow regime, the flow discharge through the friction element which has a water head of y is given by:

$$Q=Va=(ca/l)y=py \tag{2}$$

in which V is flow velocity in the friction element and ca/l is assumed to be constant and equal is to p with dimensions of L^2/T. Since the reservoir element area, a, is constant, Equation 2 shows a direct proportional relation between outflow discharge and reservoir element storage and thus it represents the linear reservoir concept. The constant head overflow device in the LAM is located above the friction element to keep the friction element always saturated. It is to be noted that the constant head overflow device is a separator in the LAM and is mathematically analogous to a unit quotient coefficient amplifier in an electric analog model (EAM) (Figure 2-b). Using the linear reservoir relation (Equation 2) and the continuity equation in the following form:

$$I - Q = \frac{dS_A}{dt} \tag{3}$$

in which Q, I and S_A are outflow, inflow and water volume of the LAM reservoir element respectively, the governing differential equation of a liquid analog circuit can be derived:

$$I - Q = \frac{dS_A}{dt} = A\frac{dy}{dt} \rightarrow A\frac{dy}{dt} + py = I \rightarrow \frac{A}{p}\frac{dy}{dt} + y = \frac{I}{p} \tag{4}$$

or

$$\frac{A}{p}\frac{dQ}{dt} + Q = I \tag{5}$$

Similarly the governing differential equation of the EAM can be derived using Krichhoff's law in the following form:

$$\left.\begin{array}{l} E_1 - E = Ri_1 \\ i_2 = C\dfrac{dE}{dt_0} \end{array}\right\} : Since \quad i_3 = 0 \;\; then \;\; i_1 = i_2 \rightarrow E_1 - E = R.C\frac{dE}{dt_0} \tag{6}$$

according to an amplifier operation with the quotient coefficient of K_0, it follows that:

$$K_0\left(E - E_2\right) = E_2 \xrightarrow{K_0 >> 0} E = E_2 \tag{7}$$

by substituting of Equation 7 in Equation 6 the final differential equation will be derived:

$$R.C\frac{dE_2}{dt_0} + E_2 = E_1 \tag{8}$$

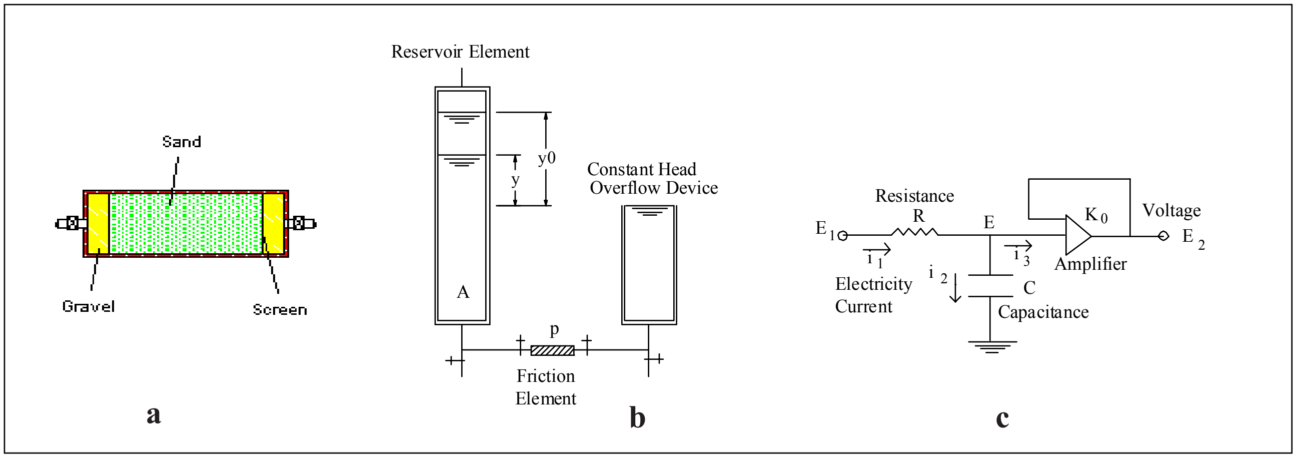

Figure 2. (a) liquid (b) electric analog circuit (c) friction element of LAM.

in which E, i, R, C, t_0 are voltage, current, resistance, capacitance and time in an electric model respectively. Like an electrical circuit in which the amplifier does not allow any current movement but voltage can move to the other side of the circuit, in a liquid analog circuit, the water head, contrary to water discharge, can not move to the next circuit due to the constant head device. Finally by combination of Equation 1 and the continuity equation, the following differential equation is derived for a conceptual linear reservoir:

$$K\frac{dO}{dT}+O=I_L \tag{9}$$

where I_L is inflow water discharge to the linear reservoir and T is time in the prototype. The analogy among the three mentioned systems can be clearly observed by comparison of Equations. 5, 8 and 9 with an arbitrary initial condition and the variables A/p, K and $R.C$ are all equivalent

In the EAM, if an annular Krichhoff's law is used instead of a nodal one, in differential Equation 8, current will be expressed as a function of t_0 instead of voltage and in this manner for the LAM in Equation 5, the water head rather than water discharge can be expressed as a function of t (Equation 4). Like the EAM, several liquid circuits can be combined in series, parallel, cascade, connected or other complex configurations to create a system with a known governing differential equation. A cascade system of two liquid circuits and its electrical analog have been shown in Figure 3. In the

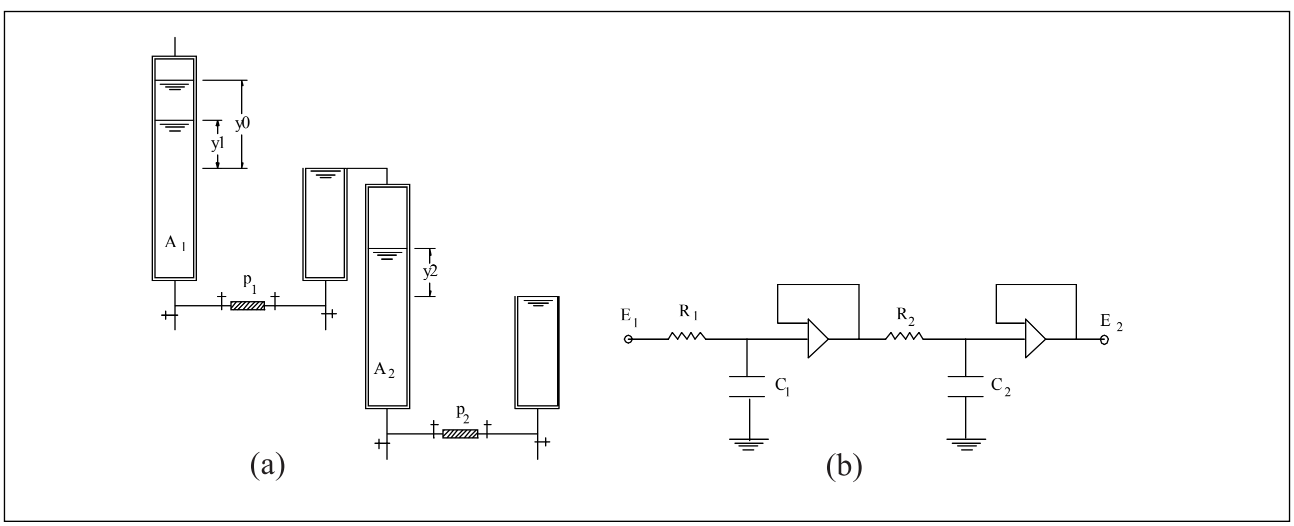

Figure 3. (a) liquid (b) electric cascade analog.

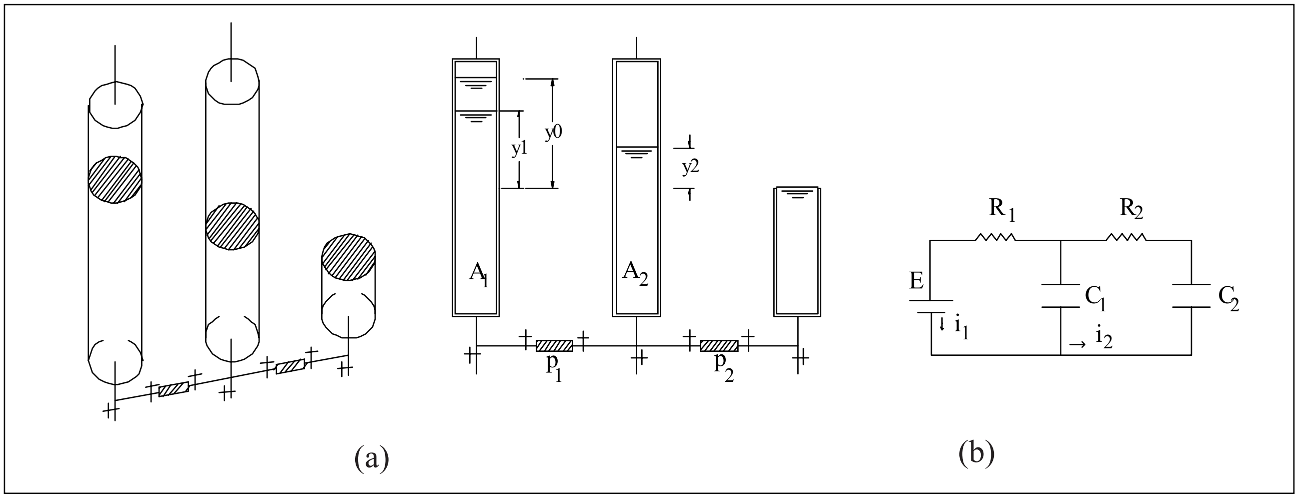

Figure 4. Connected (a) liquid (b) electric circuits.

electrical model, if the amplifiers are not used, current can move to the next circuit (Figure 4-b), similarly in the liquid system if liquid circuits (reservoir plus friction elements) are directly connected together (Figure 4-a), the water head in each reservoir will affect the other reservoir; this situation presents a linear system but with a feedback effect. Referring to Figure 4-a, if the water levels are equal in the reservoir elements, no water discharge will be transferred between reservoirs, however if an impulse as a water head or discharge is applied to one of the reservoirs, the water will flow in the system. Suppose that at time *t* the water levels in the first reservoir element with cross section A_1 is y_1 and in second reservoir element with cross section area A_2 is y_2, and the friction coefficients of the friction elements and the outflow of the reservoirs are *p1, p2, Q1,* and Q_2 respectively and $I(t)$ is input discharge to the first reservoir element, using Equations 2 and 3 the following equation set can be written:

$$\begin{cases} Q_1 = p_1(y_1 - y_2) \\ I - Q_1 = A_1 \dfrac{dy_1}{dt} \\ Q_2 = p_2 y_2 \\ Q_1 - Q_2 = A_2 \dfrac{dy_2}{dt} \end{cases} \tag{10}$$

Each of the four unknown variables (Q_1,Q_2,y_1,y_2) can be found by solving the foregoing equation set. The last output discharge (Q_2) is of more interest than the others and this variable can be expressed by the following second order ordinary differential equation:

$$\frac{d^2Q_2}{dt^2} + \left(\frac{p_1}{A_1} + \frac{p_2}{A_2} + \frac{p_1}{A_2}\right)\frac{dQ_2}{dt} + \frac{p_1 p_2}{A_1 A_2} Q_2 = \frac{p_1 p_2}{A_1 A_2} I(t) \tag{11}$$

This equation along with any initial conditions defines the behavior of the above system to produce output discharge, Q_2. The governing differential equation for a liquid cascade system (Figure 3a) will be similar to Equation 11 but without p_1/A_2 in the second term. It implies there is no interference between the reservoir elements. By a similar methodology, the governing differential equation for a LAM with *n* connected reservoir elements is:

$$[D^n+(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+...+\frac{\alpha_{2n-1}}{\beta_{2n-1}})D^{n-1}$$

$$+(\sum_{\substack{i=1\\ \alpha_i\neq\alpha_j\\ \beta_i\neq\beta_j}}^{2n-1}\sum_{j=i+1}^{2n-1}\frac{\alpha_i\alpha_j}{\beta_i\beta_j})D^{n-2}+(\sum_{i=1}^{2n-1}\sum_{j=i+1}^{2n-1}\sum_{\substack{k=j+1\\ \alpha_i\neq\alpha_j\neq\alpha_k\\ \beta_i\neq\beta_j\neq\beta_k}}^{2n-1}\frac{\alpha_i\alpha_j\alpha_k}{\beta_i\beta_j\beta_k})D^{n-3}+...+(\prod_{i=1}^{n}\frac{\alpha_i}{\beta_i})]Q_n=\prod_{i=1}^{n}\frac{p_i}{A_i}I(t) \quad (12)$$

where $D=d/dt$ is the differential operator and for

$$i=0\ \ to\ \ n: \begin{cases}\alpha_i=p_i\\ \beta_i=A_i\end{cases} \quad \text{and for } i=n+1\ \ to\ \ 2n-1: \begin{cases}\alpha_i=p_{i-n}\\ \beta_i=A_{i-n+1}\end{cases}$$

For n=2 , this equation changes to Equation 11.

Equation 12 is an ordinary linear differential equation with constant coefficients of order n. Its characteristic equation roots, which can be computed by Graeff Root Squaring method, (Wylie, 1966) will be non-zero and real non positive values, thus the system will be absolutely stable. By a similar calculation for n LAM circuits, arranged in cascade form with inflow (I_i, inflow to reservoir element i) into each reservoir element, the differential equation of the system will be :

$$\left[\prod_{i=1}^{n}\left(1+\left(\frac{A}{p}\right)_i D\right)\right]Q_n=\sum_{i=2}^{n}\left\{\left[\prod_{j=1}^{i-1}\left(1+\left(\frac{A}{p}\right)_j D\right)\right]I_i\right\}+I_1 \quad (13)$$

For $I_1=I_2=\ldots=I_n=I$ this equation is similar to the general hydrologic model differential equation with constant coefficients (McCann and Singh, 1981).

If A/p is the same LAM circuit (i.e. the physical size of reservoir elements and the properties of friction elements may be different as long as A/p remains the same for all liquid circuits and $I_1=I_2=\ldots=I_n=0$, the following equation is obtained using Equation 13:

$$\left(1+\frac{A}{p}D\right)^n Q_n(t)=I(t) \quad (14)$$

Comparing the Nash's model differential equation in the following form (Singh, 198):

$$(1+KD)^n O_n(T)=I_L(T) \quad (15)$$

and the LAM equation for n cascade circuits (Equation 14), the similarity between the two systems becomes clear. Thus, the LAM can be used for simulation of Nash's excess rainfall-direct runoff model in the laboratory (Nourani and Monadjemi, 2005).

The inputs (I_i) and output (Q_n) of the system are as functions of time (t) and the applied inputs to the system can be considered as either direct inflows or outputs of other LAM systems.

MODEL SCALING AND SET UP

Due to the simple structure and operation of the LAM, it can be easily constructed and utilized in every laboratory. The laboratory models such as the EAM (or LAM) that are used for watershed modeling and study of hydrological laws, watershed response, or creating experimental data are categorized in the laboratory prototype group and they are different from small-scale physical models (Amorocho and Hart, 1965). The LAM system when acting in a linear form, can be built and utilized for multi scaled times and discharges. When each part of a watershed is considered as a linear reservoir with storage coefficient K_i through a conceptual model, every liner reservoir can be represented by a LAM circuit with proper $(A/p)_i$ in the laboratory.

Equation 12 or 13, clearly shows that there are two independent variables, i.e. time and discharge (or head), in a liquid analog system. Therefore, two scale coefficients, time scale and discharge scale are necessary. Time scale, τ, can be computed by the following ratio:

$$\tau = \frac{(\frac{A}{p})_i}{K_i} \qquad i = 1,2,\ldots,n \tag{16}$$

and the discharge scale, γ, can be obtained by Equation 17:

$$\gamma = \frac{I_{\max}}{I_{L_{\max}}} \tag{17}$$

in which $I_{L_{\max}}, I_{\max}$ are the maximum discharge in the real system and the maximum applicable discharge in the laboratory respectively.

For the scaling and design of the LAM in the laboratory, the following procedure may be followed:

a) The conceptual model parameters (n,K_i) are estimated by any parameters estimation method.

b)A reasonable time scale, τ, is chosen.

c) From Equation 16, $(A/p)_1$ are obtained for all Lam circuits.

d) By allocating suitable values to A_i, a_i, and l_i, the values of p_i and consequently ci are determined using equation 2 e) On the basis of the values c_i, the sand in each friction element is prepared by the use of a permeability meter in a soil mechanics laboratory. It is clear that in the mentioned steps other parameters can be assumed and the rest can be obtained.

f) Discharge scale, γ, is computed by Equation 17 and then by applying this scale on input discharges of the real system, the input discharges of laboratory LAM are obtained.

g) the maximum water level in the last reservoir, *ymax is* approximately obtained by:

$$I_{\max} = p_n y_{\max} \tag{18}$$

In the model design all the parameters are selected in such a way so the reservoir elements do not overflow. The different components of the LAM constructed in the laboratory and used for the experimentation are shown in Figure 5 and a complete LAM circuit is shown in Figure 6. The calibration of individual LAM circuits which verifies the linear relation between head and discharge is strongly recommended. In this way there is no need for the discharge measurement and a simple reading of water head gives the discharge (see Equation 2). After calibration of all the circuits, the required system, having a particular configuration, will be assembled. The scaled input loads, using water, on the system in the form of pulses can be easily applied by several pumps having different capacities.

A LINEAR GEOMORPHOLOGICAL RUNOFF ROUTING MODEL

Boyd et al. (1979) developed linear and nonlinear watershed bounded network models (WBNM) of which the linear one is the subject of this paper. In this model, a catchment is divided into sub-areas bounded by watershed lines using large-scale topographic map. Therefore, the model comprises lumped storage elements, each of which represents a catchment sub-area, connected in the same arrangement as the stream network. The model structure permits the storage parameter of each element to be related to the geomorphological and hydrological characteristics of the catchment. This enables direct evaluation of several model parameters and reduces the number of parameters to be optimization, thus avoiding the problem of parameter interdependence. WBNM distinguishes between two types of sub-areas, ordered basin and interbasin area (Boyd et al., 1979). For an assumed watershed, as shown in Figure 7, the model network structure is presented in Figure 8. In the linear case, for both ordered basin and interbasin area, K_B is the lag time for the transformation of the input excess rainfall to direct runoff output. In addition, a separate lag time K_I applies to the second

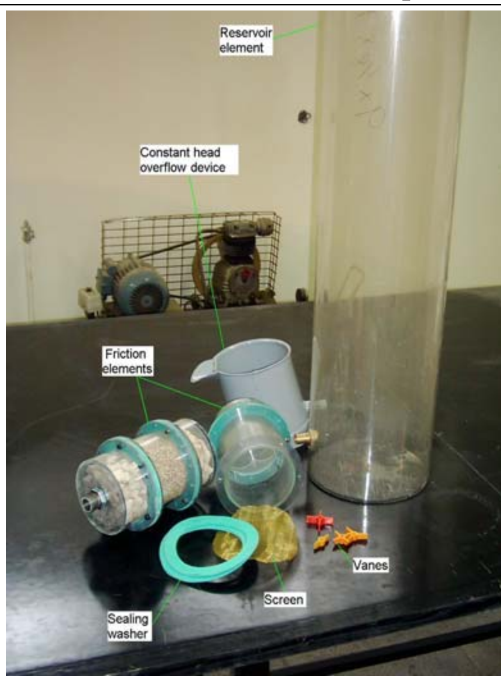

Figure 5. LAM elements and pump vanes.

Figure 6. A LAM circuit.

component of outflow from the interbasin areas resulting from upstream runoff transmitted through the stream segment. The total lag time for a network model structure with storages numbered consecutively to the outlet can be calculated by the following equation (Boyd et al., 1979):

$$T_L = \frac{\left[\sum_{m=1}^{n} A_{r_m} P_m K_{B_m} + \sum_{j} K_{I_j}\left(\sum_{m=1}^{j-1} A_{r_m} P_m\right)\right]}{\sum_{m=1}^{n} A_{r_m} P_m} \tag{19}$$

where A_{r_m} is the area of catchment sub-area represented by storage element m, P_m is the depth of rainfall excess on element m and j represents only those sub-areas that are interbasins. For elements in series with inflow to the top element only, and all elements of equal K, Equation 19 reduces to the Nash model so:

$$T_L = nK \tag{20}$$

A network with n' ordered basins can be realistically considered as $2n'$ -1 parallel paths in a cascade formation which the rainfall takes to arrive at the outlet of the watershed. Therefore the IUH of the model will be the summation of these cascades (Singh, 1988):

$$h(T) = \frac{1}{A_T}\sum_{i=1}^{2n'-1} \frac{1}{(1+K_{B_i}D_L)\prod_{j_1}(1+K_{I_{j_1}}D_L)}\delta(T)A_{r_i} \tag{21}$$

where A_T is the area of whole basin, A_{r_i} is the contributed of the i cascade, n^n is the number of interbasin areas in the i cascade and $D_L = d/dT$. In Figure 9 all possible paths of the flow, for the watershed shown in Figure 7, are shown. Boyd et al. (1979) related the storage coefficients and sub-basins area through power functions:

$$K_{B_i} = a_{G_1} A_{r_i}^{b_1} \tag{22}$$

and

$$K_{I_i} = a_{G_2} A_{r_i}^{b_2} \tag{23}$$

where the unit of K_B, K_I is hours and the unit of A_r is km^2. The constant coefficients B_2, b_1, a_{G_2}, a_{G_1} are derived by statistical analysis. In the current study, for simplification of the model, it is assumed that the ordered basins coincide with the interbasin areas i.e. $K_{B_i} = K_{1_i} = K_i$. Thus in this case Equation

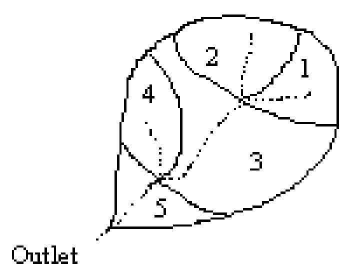

Figure 7. A watershed divided to sub-basins.

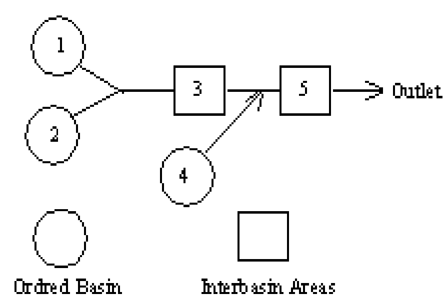

Figure 8. Network representation of the watershed.

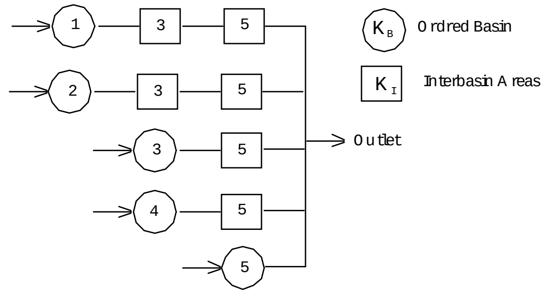

Figure 9. Cascade representation of the watershed.

21 simplifies to the following equation using the IUH equation for a cascade including different linear reservoirs (Singh, 1988):

$$h(T)=\frac{1}{A_T}\sum_{i=1}^{2n'-1}\left(\sum_{j_2}\frac{K_{j_2}^{n'-2}e^{-T/K_{j2}}}{\prod\limits_{\substack{i'=1\\ i'\neq j_2}}\left(K_{j_2}-K_{i'}\right)}\right)A_{r_i} \qquad \text{For more than one reservoir in } i \text{ th cascade} \tag{24}$$

and

$$h(T)=\frac{A_{r_i}}{A_T}\frac{1}{K_i}e^{-T/K_i} \qquad \text{For only one reservoir in } i \text{ th cascade} \tag{25}$$

in which i', j_2 denote interbasin areas in the i th cascade and n″ is the number of interbasin areas in the i th cascade. As an example, for the assumed watershed shown in Figure 7 and represented by five paths (Figure 9), the basin lag time can be written in the following form using Equation 19 for a constant rainfall over the basin:

$$T_L=\frac{\left[\left(\sum_{i=1}^{5}A_{r_i}K_i\right)+A_{r_1}K_3+A_{r_2}K_3+A_{r_1}K_5+A_{r_3}K_5+A_{r_2}K_5+A_{r_4}K_5\right]}{A_T} \tag{26}$$

To compute the IUH, Equations 24 and 25 can be used, for example the second path contributes in the total IUH as follows:

$$h_2(T)=\frac{A_{r_2}}{A_T}\left(\frac{K_2e^{-T/K_2}}{(K_2-K_3)(K_2-K_5)}+\frac{K_3e^{-T/K_3}}{(K_3-K_2)(K_3-K_5)}+\frac{K_5e^{-T/K_5}}{(K_5-K_2)(K_5-K_3)}\right) \tag{27}$$

and the fifth path as follows:

$$h_5(T)=\frac{A_{r_5}}{A_T}\frac{e^{-T/K_5}}{K_5} \tag{28}$$

evidently the total IUH of the watershed will be the summation of h_1, h_2, h_3, h_4, h_5 .

RESULTS AND DISCUSSION OF THE MODEL APPLICATION

The proposed LAM was used in the laboratory in order to evaluate its efficiency in simulation of Boyd's model. For this purpose April 25-26, 2001 storm data of Tajyar watershed with the area of 128 km^2 and mean slope of 27% at Sarab, East Azerbaijan, Iran were used. The main channel of the Tajyar has a length of 35 km with 3.7% net slope and lag time about 7 hours. As shown in Figure 10, the Tajyar is one of the main branches of the Ajichai river.

The measured rainfall at the Sarab meteorology station, is assumed constant over the basin, and the Tajyar observed output hydrograph at the Mirkoh hydrometry station are presented in Table 1. The direct output hydrograph was computed by subtraction of the base flow (O_0=2.74 m^3 /s)from the observed hydrograph. The excess rainfall hyetograph obtained by applying the ϕ–index method ($\phi\Delta T$=0.613 mm/.5h) considering that any rainfall prior to the beginning of direct runoff is taken as initial abstraction (Chow et al., 1988), which is assumed to be a small amount in the current example. The calculated excess rainfall hyetograph and direct output hydrograph are shown in Figure 11. The watershed was divided into five sub-areas (n=5) and the area of each sub-basin was derived (Table 2) using large-scale topographic map. The schematic representation of the watershed and its flow paths will be just like the mentioned example in the last section, which are shown respectively by Figures 7 to 9. Every sub-area was represented by a linear reservoir with storage coefficient related geomorphologically to the area through a power relationship:

$$K_i = a_0 A_{r_i}^{b_0} \tag{29}$$

in this relation usually b_0 has a limit range but a_0 is widely variable from one region to another

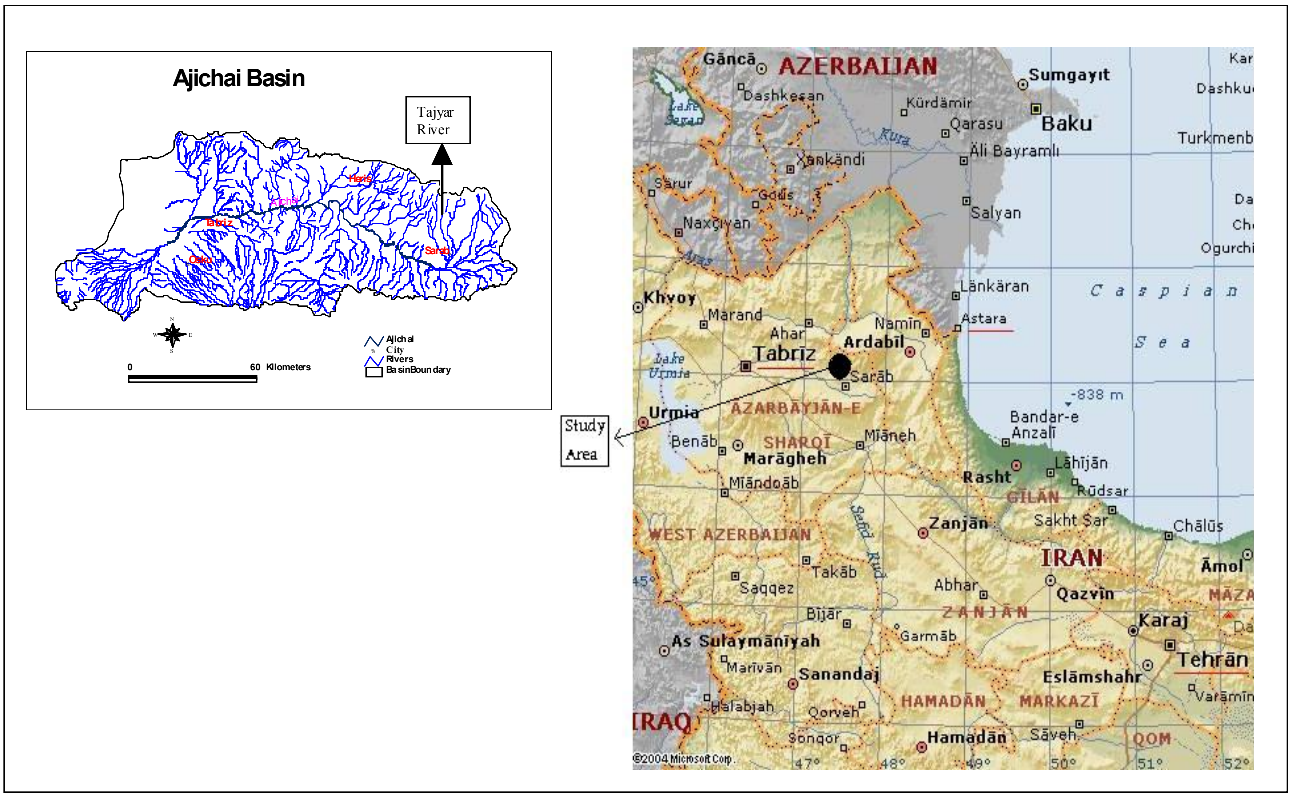

Figure 10. Study Area.

Table 1. Observed Rainfall and Runoff of Tajyar Watershed

Observed Runoff (m^3/s)	**Time**(hour) 26 Apr. (2001)	**Rainfall Hyetograph** (mm/.5h)	**Time**(hour) 25-26 Apr. (2001)
2.2	Before 0	0.4	21-21.5
7.74	0-2	0	21.5-23
11.93	2-4	0.6	23-23.5
14.25	4-6	0	23.5-24
12.54	6-8	2.8	24-0.5
10.84	8-10	1.5	0.5-1
8.89	10-12	0	1-
7.63	12-14		
2.74	14-16		
2.74	16-18		

(Singh, 1988). In the present study b_0 was considered equal to 0.38 (Boyd et al., 1979) but a_0 was computed using lag time. Considering that lag time is equal to the time from the centroid of the excess rainfall hyetograph to the centroid of the direct runoff hydrograph, the whole basin lag was obtained, T_L=6.8 hrs, which is approximately equal to the reported lag time (i. e. 7 hrs). Consequently, by substituting sub-basin areas (Table 2) into Equation 26 and using Equation 29, a_0=0.825 was obtained for the current example. Thus all parameters (n,K_i) were directly obtained without any optimization and are presented in Table 2.

It must be noticed although T_L was computed using storm data, it can be considered as a geomorphological characteristic of the watershed and related to the watershed physical properties through some mathematical equations without any calibration (Singh, 1988). The model IUH was found by Equations 24 and 25 as:

$$h(T) = 2.1e^{-T/3.7} - 4e^{-T/3} + 0.35e^{-T/2.1} - 0.17e^{-T/2.14} + 1.72e^{-T/2.7} \tag{30}$$

with inverse hours units and shown in Figure 12.

The linear assumption for any watershed hydrologic system yields a direct runoff given by the convolution integral in the following matrix form (Singh, 1988):

$$[O]_{N\times 1} = [I_L]_{N\times(N-M+1)} \cdot [h.\Delta T]_{(N-M+1)\times 1} \tag{31}$$

Table 2. Sub-Basin Characteristics

Rainfall to Sub-basin(m^3/s)		Storage Coefficient (hour)	Sub-basin Area (km^2)	Sub-basin No.
Second Pulse	First Pulse			
25.3	62.5	3.7	51.37	1
5.8	14.3	2.1	11.7	2
14.5	35.8	3	29.41	3
6.2	15.3	2.14	12.52	4
11.2	27.7	2.7	22.72	5
Total rainfall discharges to the whole basin were computed 155.6 m^3/s, 63 m^3/s for the first and second pulses respectively.				

where M and N are the data numbers of rainfall and runoff respectively and $[I_L]$ is a Toeplitz band matrix that can be obtained by rainfall data. For the example at hand: N=28, M=2, N-M+1=27, ΔT=30 minutes. To evaluate the general IUH of a watershed, many storm data of the watershed must be used to verify the IUH; but regarding the purpose of the current study, which is to demonstrate the educational aspects of the LAM, only one storm event was used. The 30 minutes direct runoff hydrograph was predicted using the model IUH (Equation 30) and Equation 31 and then converted to the 2 hour hydrograph.

For more comparison, another procedure was done to compute the IUH, so that the reservoir storage coefficients were computed by the following relations insisted of by Equation 29:

$$K_1 = a_{0_1} A_{r_1}^{b_0}, \quad K_2 = a_{0_1} A_{r_2}^{b_0}, \quad K_4 = a_{0_1} A_{r_4}^{b_0}, \quad K_3 = a_{0_3} A_{r_3}^{b_0}, \quad K_5 = a_{0_5} A_{r_5}^{b_0} \tag{32}$$

where b_1=0.38 and $a_{0_1} = 0.6$, $a_{0_3} = 0.7$, $a_{0_5} = 1.7$ were estimated by calibration using the storm data.

The computed direct runoff using the two methods mentioned and observed direct runoff along with the model fitting criteria (N_e, Nash and Sutcliffe, 1970) are shown as two scatter plots in Figure

13. The results show increasing degrees of freedom does not increase the model accuracy very much, but it is expected that the model is affected by the number of linear reservoirs and the network structure. Regarding this matter, Boyd et al. (1979) stress the network configuration has more influence than the number of the storages for the model goodness of fit. In any case; because the first model was established without calibration, it is hoped that the model efficiency criterion (N_e) will be the same for other storm events in which this value for N_e is not low in the model verification procedure; therefore the first model was chosen for Tajyar watershed. The continuity condition was not completely satisfied for computed direct runoff hydrograph, because according to the IUH diagram (Figure 12), the whole IUH was not contributed to create the direct runoff in the storm duration. The input rainfall discharges to the sub-basins as two pulses are presented in Table 2.

To design the LAM system for Tajyar watershed, a suitable time scale was selected , the $(A/p)_i$ for each LAM circuit was found by Equation 16, reservoir section area for each reservoir element, A_i , was chosen, then using the obtained value of p_i ,the friction element characteristics were computed. I_{Lmax} and I_{max} were chosen according to the rainfall data (Table 2) and the pump capacities respectively and discharge scale were obtained by Equation 17. The characteristics of the model elements and the scale coefficients are shown in Table 3.

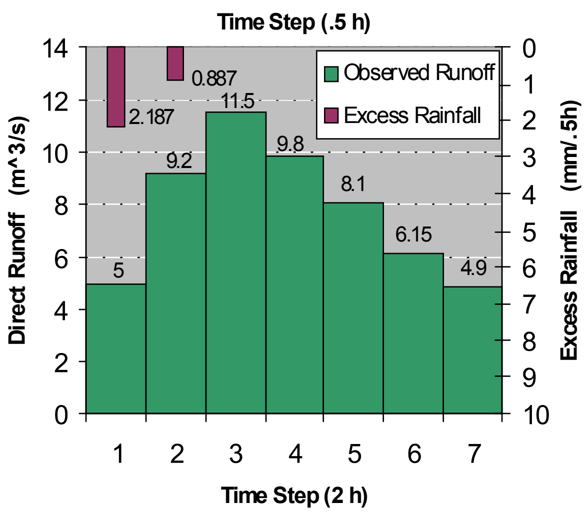

Figure 11. Excess rainfall direct runoff

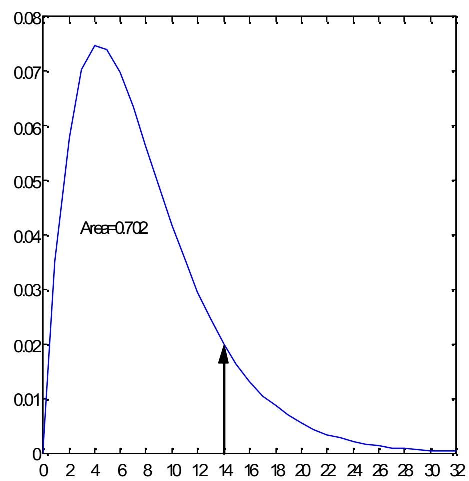

Figure 12. IUH of Tajyar watershed

After calibration of the LAM circuits, they were assembled to simulate Boyd's model for the Tajyar watershed in the laboratory as shown in Figure 14. The prescribed scaled input discharges were applied to the reservoir elements in the form of two consecutive pulses and in the scaled time duration (each scaled time duration is Δt in the laboratory) by the pumps. The scaled output discharges of the system were obtained by measuring the water levels at the last reservoir element and transforming to the discharges using Equation 2. Finally by applying discharge scale coefficient on the measured discharges, the watershed output hydrograph was computed (Table 4). The computed direct runoff hydrographs using Boyd's theoretical and liquid analog models along with the fitting goodness criterion (R^2) have been shown in Figure 15 as a scatter plot. It is seen by comparison of the obtained two hydrographs (Table 4 and Figure 13-a), for high discharges, i.e. for high water heads in the last reservoir element, the computed values in the LAM system depart from theoretical Boyd's model. Such deviations are thought to be due to the system head losses. Such head losses become more pronounced with increasing V.

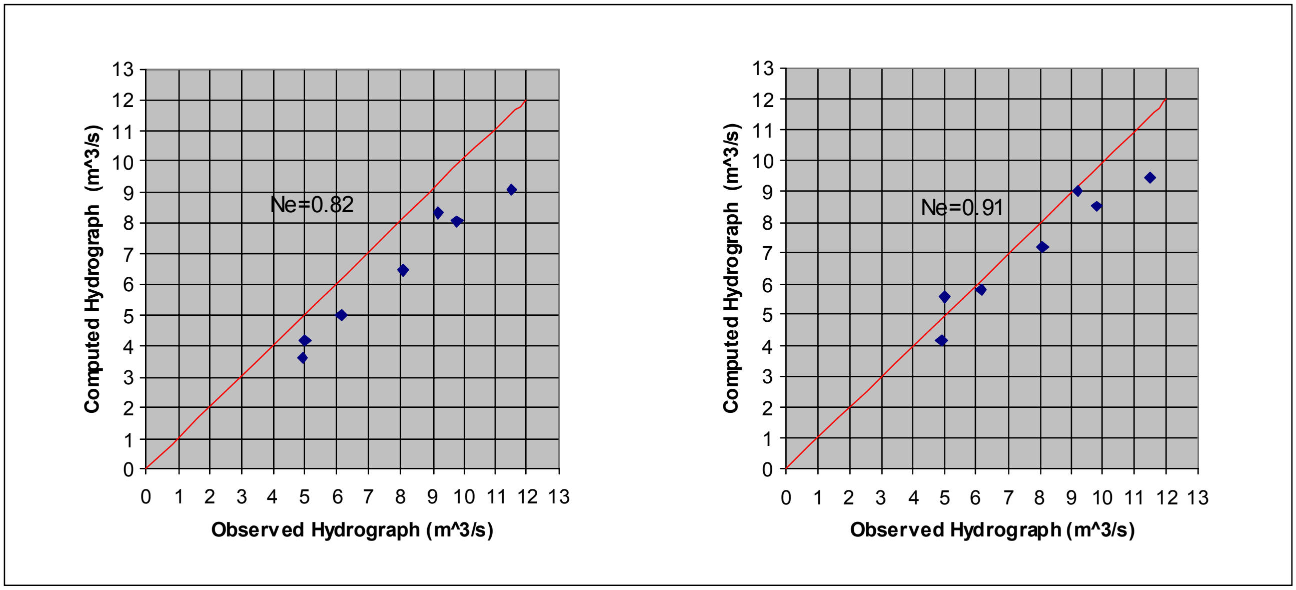

Figure 13. Observed and computed hydrographs (a) without (b) with calibration.

In addition to water head reading and head losses errors, another error which is related to the pumps operation, as input signal noises, affected on the system output, especially in the initial time.

It is also possible to simulate both interbasin and ordered sub-areas separately by the LAM, instead of assuming that these reservoirs coincide; scaled input pulses must be applied only to the ordered sub-basins. However, in this case the laboratory LAM needs two more circuits and consequently more space and cost.

Furthermore, the LAM may be reliably used for simulation of some other hydrological routing problems such as the effect of watershed shape on the output hydrograph and the response of a watershed to the storm movement direction (Nourani et al., 2005).

The governing equations of the liquid analog system, in the form of either cascade or connected, are analogous to some other engineering phenomena such as ground water flow, river pollution, etc. Therefore the LAM can be adequately applied for laboratory simulation of similar problems as the EAM; also the LAM can be made and operated in a nonlinear mode, by using a cone as the reservoir element as well. The details of these subjects are left to future papers.

The LAM system operation in the laboratory possesses several advantages including:

1. It can be easily constructed by simple components.

2. It can be easily operated by non-expert and non-professional persons in any laboratory.

3. It can be a very useful instrument in the field of flow routing in hydrology education because of its visible properties and operation (Nourani et al., 2005).

4. The LAM has been built on the basis of simple hydraulics laws so this model is more understandable than the EAM, especially for civil engineering students.

5. Time required for each experiment is more appropriate in contrast to the EAM which is too fast. Therefore the liquid analog system can be observed, analyzed and contemplated in a more appropriate way.

6. It is possible to stop the flow process in the LAM if required and start it again with no loss of continuity in data gathering.

Table 3. LAM and Input Load Characteristics

Input to LAM** (lit/min)		**Reservoir element section area(A)** (m^2)	**Hydraulic conductivity of friction element(c)** (m/s)	**Friction element section area(a)** (m^2)	**Length of friction element*(l)** (m)	**Circuit No.**
pulse 2 nd	pulse 1 th					
4.05	10	0.038	0.0054	0.0095	0.165	1
0.93	2.3	0.0314	0.0054	0.00785	0.095	2
2.3	2.7	0.038	0.0054	0.00785	0.11	3
1	2.45	0.038	0.0054	0.00785	0.08	4
1.8	4.43	0.038	0.0054	0.0095	0.12	5

*According time scale ($\tau = \frac{1}{109}$) l_i were computed by assuming other variables using Eq. 16.

**Each pulse in laboratory was $\Delta t = \frac{\Delta T}{109} = \frac{1800}{109}$ s

- $I_{L_{\max}} = 62.5\ m^3/s$ in prototype and $I_{\max} = 10\ lit/\min$ in the laboratory so discharge scale was $\gamma = \frac{1}{375000}$

Figure 14. Constructed LAM in laboratory for Tajyar watershed.

On the other hand there are some disadvantages in the LAM including:

1. More space and cost needs compared to the EAM.

2. Less accuracy due to the existence of head losses and other experimental errors in comparison with the EAM.

3. Less flexibility than the EAM, because some elements of the EAM have not been yet simulated for the LAM.

It is hoped that the LAM system for runoff simulation will be used as a laboratory device in the future.

Table 4. LAM System Output for the Tajyar Watershed

Output Hydrograph of LAM* (m^3/s)	**Output Discharges of LAM** (lit/min)	**Time** (2 hours)
5	0.8	1
9.2	1.472	2
11.5	1.84	3
9.8	1.568	4
8.1	1.296	5
6.15	0.984	6
4.9	0.784	7
*Obtained by applying γ on column two.		

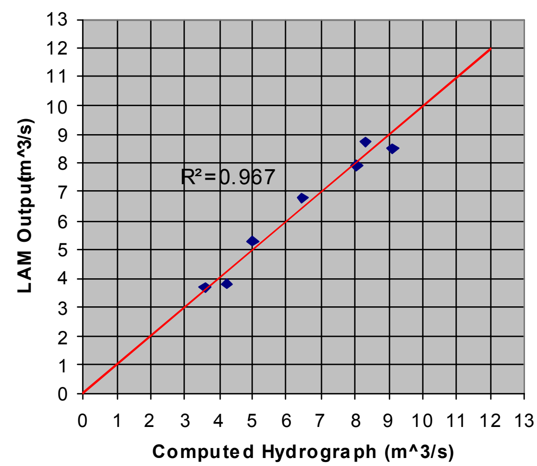

Figure 15. LAM output and computed hydrograph.

CONCLUSION

The Liquid Analog Model has been built on the basis of continuity and Darcy's laws. It can be reliably used for laboratory simulation of not only the hydrological routing models but also some other engineering systems. This model involves the linear reservoir concept and each liquid analog circuit has three major components; the first is a reservoir element, the second is a friction element as a porous medium, and the third is a constant head overflow device.

To verify the proposed model it was successfully used to simulate an actual watershed response for a real storm event in the laboratory based on Boyd's geomorphological model. For this purpose the circumstances of carrying out the LAM scaling and its construction in the laboratory were extensively described.

According to the properties and abilities of the LAM, it could be demonstrated that it constitutes a potentially valuable tool in hydrologic investigation and education. Of even greater importance is the fact that it facilitates observation of the fundamental behavior of a hydrologic system. It would be gratifying also if one could gain certain insight into some of the significant parameters that could be correlated universally with the physical characteristics of a drainage basin by using the LAM. Consequently, regarding the study described herein, the Liquid Analog Model can be added to the other hydrological models as shown in Figure 1.

For more perception of the LAM capabilities, it is suggested to apply it for modeling some other hydrological problems in both of cascade and connected configurations.

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